

Adaptive Iterative Learning Control of a Single-Link Flexible Manipulator Based on an Identified Adaptive NARX Model

Thesis submitted to
National Institute of Technology, Rourkela
For award of the degree

of

Master of Technology

in

Control and Automation

by

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Department of Electrical Engineering
National Institute of Technology Rourkela
May 2013

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Certificate

This is to certify that the thesis entitled “**Adaptive Iterative Learning Control of a Single-Link Flexible Manipulator Based on an Identified Adaptive NARX Model**” by **Dinesh Mute**, submitted to the National Institute of Technology (NIT), Rourkela for the award of Master of Technology in **Control and Automation**, is a record of bona fide research work carried out by him in the Department of Electrical Engineering, under our supervision and guidance.

I believe that this thesis fulfills part of the requirements for the award of degree of Master of Technology. The results embodied in the thesis have not been submitted for the award of any other degree elsewhere.

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Dinesh Mute
Rourkela, May 2013

Abstract

Flexible-link robots are more widely used in areas such as microsurgery, defense and space vehicles. Compared to rigid robot manipulator because of light weight, low inertia, higher payload carrying capacity and faster executable motion. Flexible robots consist of manipulators that are made of flexible and lightweight materials. In addition to these benefits they are associated with serious control problem of vibration. As the structure is flexible when it is actuated and it vibrates with low frequency and after some time the vibrations gets reduced. Hence the control problem for the flexible robot becomes much more complex than rigid link robots. So to overcome this difficulty many control strategies have been proposed in the past, but in most of the works they have considered linear model and the actuator dynamics are not taken into account.

In this project work, a non linear model of a single link flexible robot manipulator obtained using Assumed Mode Method (AMM) is considered. The actuator dynamics has also been considered in the modelling of the single link flexible robot. The model obtained by using AMM method is quite complex and require complete knowledge of flexible manipulator dynamics. As the system is nonlinear and time-varying so to meet the demands of the control system design, an adaptive nonlinear autoregressive with exogenous input (NARX) model is identified using the input/output experimental data. An adaptive iterative learning controller (AILC) is designed based on both the existing mathematical model and identified model of a single-link flexible manipulator (SLFM). Tuning of the AILC controller is carried out using least square method. Simulation results demonstrate that the AILC controller designed using the identified adaptive NARX model gives better results than the AILC controller designed using an existing mathematical model.

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List of Acronyms

List of Acronyms

FRM	:	Flexible Robot Manipulator
AMM	:	Assume Mode Method
FEM	:	Finite Element Method
OE	:	Output Error
ARX	:	Autoregressive with Exogenous input
NARX	:	Nonlinear Autoregressive with Exogenous input
ILC	:	Iterative Learning Control
AILC	:	Adaptive Iterative Learning Control
P	:	Proportional
PD	:	Proportional Derivative
LS	:	Least Square

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Chapter 1

Introduction to Flexible Link Robot Manipulator

The Chapter is organized as follows. Section 1.1 gives a brief introduction of a flexible-link robot manipulator and highlighting its applications, advantages and control complexities associated with these robots are also illustrated. Section 1.2 presents a literature survey on the modelling, identification and control strategies of flexible-link robot manipulators. In Section 1.3. Objective of the thesis are presented. Finally, the outline of the thesis is presented in Section 1.4.

1.1 Introduction

1.1.1 Flexible Link Robot Manipulator

Flexible-link robots are more widely used in many areas such space vehicles, defense, microsurgery. Compare to rigid robot manipulator, flexible link robot offers the following advantages : light weight, low inertia, higher payload carrying capacity and faster executable motion. Flexible robots consist of manipulators that are made of flexible and lightweight materials. The flexible robots are not only lighter than conventional rigid robots but they are also fast in response. However, in addition to these benefits they are associated with serious control problem of vibration. As the structure is flexible when it is actuated it vibrates with low frequency and after some time the vibrations gets reduced. Therefore the control problem for the flexible robot becomes much more complex than rigid link robots.

1.1.2 Advantages of Flexible Link Robot Manipulator

The various advantages of flexible robots are given below:

- Lightweight
- Faster Response
- High payload-to-arm weight ratio
- Lower power consumption

1.2 Literature Review

1.2.1 Review on modelling of a Flexible Link Robot Manipulator

The modelling and control aspects are reviewed by Dwivedy in [1]. The dynamic model of flexible manipulator is based on the Bernoulli-Euler beam theory [2], in which the dynamics are represented in the form of a partial differential equation. The final dynamical model of flexible link robot manipulator is given by finite element method in [2][3] and assumed mode method in [6][7]. Hastings and Book [4], Wang and Vidyasagar [5] gives a linear dynamic model and the transfer function of single-link flexible manipulators using Lagranges equation and the assumed mode method. A complete non linear model for single link flexible manipulator using assumed mode method is presented by Luca and Siciliano in [6]. In their work they reported about different modes of vibration . The Luca and Siciliano is extended for two link flexible manipulator [7] . However, explicitly representing the distributed model using two finite modes as in case of assumed mode method or with two to three elements as considered in the case of finite element, measurement resulted only in the approximate result, with the additional requirement of precise signal measurement. To avoid this, Ge, Lee and Zhu [8] proposed an implicit partial differential equation (PDE) model of single link flexible robot with design of a simple controller using strain gauge measurement. Ahmad et.al [9] represented the dynamic model of a two link flexible manipulator by incorporating payload. However the actuator dynamics are not considered. Subudhi and Morris [10] derived the dynamical model by using assume mode method and also gives the singular perturbation model of a two link flexible robot manipulator with under actuated flexible link and joint. The model given above is quite complex and require complete knowledge of flexible manipulator dynamics. To overcome this difficulty we go for identification of flexible link robot manipulator.

1.2.2 Review on Robot Identification of a Flexible Link Manipulator

For estimation of a flexible link robot manipulator model different identification methods have been applied. Rovner and Connon [11] used an autoregressive moving average (ARMA) model for represent the single-link flexible manipulator, and also in [12] an ARMA model with weighted recursive least square (RLS) algorithm is adopted for parameter estimation. Yurkovich and Tzes [13] presented an identification and control of a single-link flexible manipulator using on-line frequency domain linear model. A major limitation with the above techniques is the assumption of linear dynamics in the model development. In this required, in this work, the model is identified considering nonlinear and time-varying dynamics.

1.2.3 Review on Controller design for a Flexible Link Robot Manipulator

The applicability of ILC controller for flexible manipulator has been exhibited in [17-18]. As the flexible robot manipulator are generally used in repetitive task, in literature [17-18] it is shown that ILC technique gives better results for repetitive process. It enhances the tracking performance from operation to operation. The ILC techniques are developed both for linearized model or nonlinear model, with a certain a priori knowledge of the system dynamics. Hence we go for ILC controller technique based on an identified adaptive NARX model.

1.3 Thesis Objectives

The objectives of this thesis are as follows:

- To achieve precise tip-tracking of a single link flexible robot manipulator undergoing vibrations.
- To study the dynamics of a flexible link robot manipulator by using Assumed mode method (AMM).
- To identify the flexible link robot manipulator from experimental data using an adaptive NARX model.
- To design Iterative Learning control technique based on an identified adaptive NARX model.
- To make learning gain adaptive for designing an adaptive iterative learning controller.

1.4 Thesis Outline

The thesis is organized as follows.

- Chapter 1, provides brief introduction of flexible robots, their applications, their advantages, review of past works and control complexity.
- Chapter 2, presents the system descriptions and the dynamic modelling of a Single Link Flexible Manipulator by using assumed mode method and also presents the dynamics of actuator.
- Chapter 3, presents the introduction of system identification and identification of Single Link Flexible Manipulator by using different types of model.
- In chapter 4, the designing of Controller for a Single Link Flexible Manipulator is presented. This is followed by simulation results and discussion.
- Chapter 5, concludes the thesis and future scopes for further work.

Chapter 2

Dynamic Modelling of a Flexible Robot Manipulator system

In this chapter, Dynamic modelling of a flexible robot manipulator is discussed. Before going for modelling, a small introduction about experimental setup of flexible robot manipulator is presented in section 2.1. In section 2.2, derivation of closed-form dynamic model of a flexible robot manipulator is given in details. Finally the complete dynamic model of FRM with including DC motor dynamics is presented in section 2.3.

2.1 System Description

The experimental setup of a two-Link Flexible Robot Manipulator is depicted in Fig.2.1. This robot system consists of several components such as Q8 terminal board, linear amplifier, Data acquisition board and different sensors like strain gauge, quadrature optical encoder, limit switches etc. There are two links and the links are flexible and instrumented with strain gauges. There are two hub or joints of the system where strain gauges are attached to measure tension in the link. The two flexible links are actuated by dc motor installed with strain gauges at the clamped end of the links for measurement of tip deflection. We provide the input signal in the form of torque which is produced by DC motor to the flexible manipulator system to drive the link of manipulator. The output of the system is taken as a joint angle rotation of a link, which is measured by sensors.

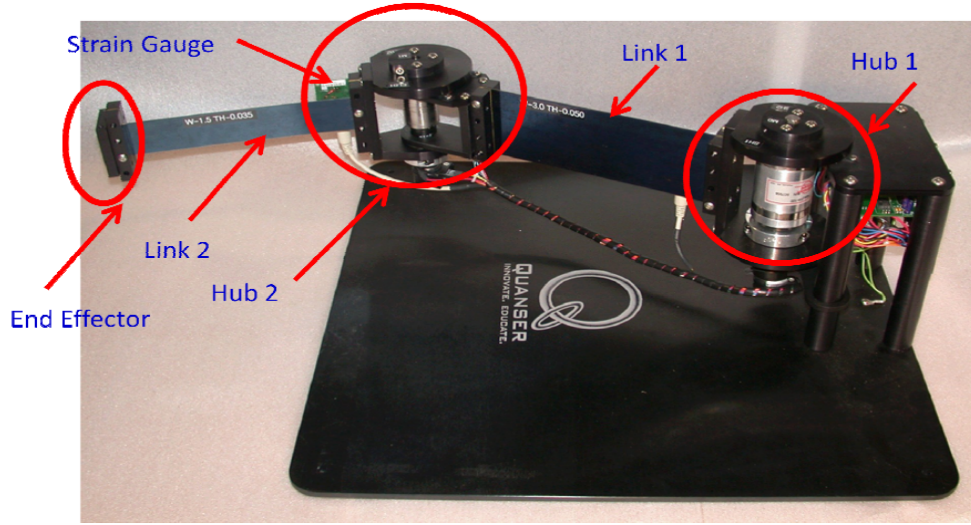


Figure 2.1: Experimental setup of a two-link flexible robot manipulator

2.2 Dynamics of flexible robot manipulator

2.2.1 Dynamic modelling of a single link flexible robot manipulator

1. To describe the dynamics of a flexible robot the Partial Differential Equation(PDE), known as Eulers Lagrange equation is used.
2. In this project, Assume Mode Method is used for modeling.

Before modelling of the single link flexible robot, we need to consider following assumptions for the link:

1. The flexible link of the robot is an Euler Bernoulli beam with uniform density.
2. The deflection in the beam is small compared to its length
3. The payload mass attached is a concentrated mass
4. The Flexible link manipulator operates in horizontal plane.
5. Thickness of the beam is small compared to its free length.

Figure (2.2) shows a single-link flexible robot manipulator under vibration. Some notations used in deriving the model of the single-link flexible manipulator system are given as follows:

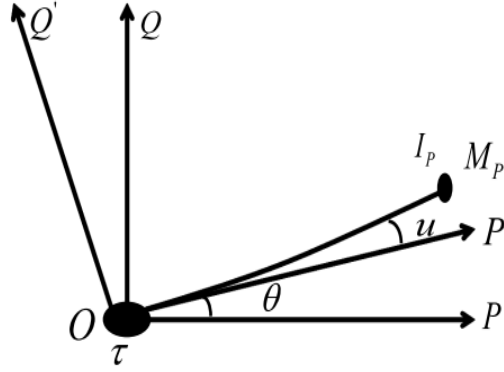


Figure 2.2: Deflection of a single flexible link

$y = (x, t)$: Position of the tip measured from reference ie. the P-axis in Figure(2.2).

$u = (x, t)$: Elastic deflection of a beam measured from undeformed structure.

l : Length of the flexible link.

M_P : Payload mass attached to the tip.

ρ : Uniform linear mass density in kg/m.

I_h : Hub inertia.

$\tau(t)$: Torque applied by the motor to the hub.

$\theta(t)$: Joint rotation angle of the beam.

EI : Uniform flexural rigidity of the beam.

Displacement $y(x, t)$ of a point along the link of robot at a distance x from the hub is given as:

$$y(x, t) = x\theta(t) + u(x, t) \quad (2.1)$$

To obtain dynamic equations of the motion of manipulator, the energies associated with manipulator link needs to be obtained. The links are modeled by using Euler-Bernoulli beam theory as discussed in following section.

The total kinetic energy of the system is given by the sum of following components:

$$KE(E_k) = \text{KE at hub} + \text{KE of link} + \text{KE at tip point.}$$

$$E_k = \frac{1}{2}I_H\dot{\theta}^2 + \frac{1}{2}\int_0^l \left[\frac{\partial u}{\partial t} + x\dot{\theta}\right]^2 \rho dx + \frac{1}{2}M_P\left[\frac{\partial u}{\partial t} + x\dot{\theta}\right]_{x=l}^2 \quad (2.2)$$

The potential energy (without considering gravity i.e horizontal plane motion) is given by

$PE(E_p) = PE$ due to the elastic deformation of the link.

$$E_p = \frac{1}{2}EI \int_0^l \left[\frac{\partial^2 u}{\partial t^2} \right]^2 dx \quad (2.3)$$

The work done for a given input torque (τ) is

$$W = \tau\theta \quad (2.4)$$

The Euler-Bernoulli equation for a beam is represent in the form of partial differential equation as follows:

$$(EI) \frac{\partial^4 u(x,t)}{\partial x^4} + (\rho) \frac{\partial^2 u(x,t)}{\partial t^2} = -\rho x \ddot{\theta} \quad (2.5)$$

where, (ρ) is the uniform mass density and (EI) is the constant flexural rigidity and $u(x,t)$ is the deformation. using equation (2.1) and (2.5), we get

$$(EI) \frac{\partial^4 y(x,t)}{\partial x^4} + (\rho) \frac{\partial^2 y(x,t)}{\partial t^2} = 0 \quad (2.6)$$

For solving this equation proper boundary conditions is applied at the base and at the end of link. At the base end the associated boundary conditions are given by

$$y(0,t) = 0, \quad y'(0,t) = 0 \quad (2.7)$$

These two boundary conditions are signifies that there is no deflection at the base end. However, the payload mass attached to the tip contributes the inertia and moment, so the boundary condition at the free end are represented as

$$M_p \frac{\partial^2 y(l,t)}{\partial x^2} - (EI) \frac{\partial^3 y(l,t)}{\partial x^3} = 0, \quad I_p \frac{\partial^3 y(l,t)}{\partial t^2 \partial x} + (EI) \frac{\partial^2 y(l,t)}{\partial x^2} = 0 \quad (2.8)$$

The solution of the dynamic equation of motion (equation 2.6) for the tip position of the manipulator can be obtained by using assumed modes method (AMM). It is represent as a linear combination of the product of admissible functions ϕ_i and ∂_i are as follows:

$$y(x,t) = \sum_{i=0}^n \phi_i(x) \partial_i(t) \quad (2.9)$$

where, n is the number of modes of vibration. $\phi_i(x)$ is the mode shape associated with i^{th} modes of link vibration and it is a function of displacement along the length of the flexible

manipulator and $\partial_i(t)$ is the generalized co-ordinates of the beam, and it is a function of time. Two modes of link vibration are enough to represent the tip deflection. Substituting equation (2.9) in equation (2.9), results in two ordinary differential equations, given as:

$$\frac{d^4 \phi_i(x)}{dx^4} - \beta_i^4 \phi_i(x) = 0, \frac{d^2 \partial_i(t)}{dt^2} + w_i^2 \partial_i(t) = 0 \quad (2.10)$$

where, $w_i^2 = \frac{EI}{\rho} \beta_i^4$

Solving equation (2.10), we get the expression for the mode shapes and the generalized coordinates:

$$\partial_j(t) = \exp(jw_j t) \quad (2.11)$$

$$\phi_j(x) = C_{1,j} \sin(\beta_j x) + C_{2,j} \cos(\beta_j x) + C_{3,j} \sinh(\beta_j x) + C_{4,j} \cosh(\beta_j x) \quad (2.12)$$

where, $\beta_j^4 = \frac{w_j^2 \rho}{EI}$ and w_j is the j^{th} natural angular frequency of the link undergoing vibration. Applying first boundary conditions ,we get

$$C_{3,j} = -C_{1,j} \text{ and } C_{4,j} = -C_{2,j} \quad (2.13)$$

and by applying second boundary conditions yields a 2×2 matrix equation

$$\begin{bmatrix} F_{1,j} & F_{1,j} \\ F_{2,j} & F_{2,j} \end{bmatrix} \begin{bmatrix} C_{1,j} \\ C_{2,j} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2.14)$$

where,

$$F_{11} = -\beta_j^2 \sin(\beta_j L) - \beta_j^2 \sinh(\beta_j L) - \frac{J_L}{\rho} \beta_j^5 \cos(\beta_j L) + \frac{J_L}{\rho} \beta_j^5 \cosh(\beta_j L)$$

$$F_{12} = -\beta_j^2 \cos(\beta_j L) - \beta_j^2 \cosh(\beta_j L) + \frac{J_L}{\rho} \beta_j^5 \sin(\beta_j L) + \frac{J_L}{\rho} \beta_j^5 \sinh(\beta_j L)$$

$$F_{21} = -\beta_j^3 \cos(\beta_j L) - \beta_j^3 \cosh(\beta_j L) + \frac{M_L}{\rho} \beta_j^4 \sin(\beta_j L) - \frac{M_L}{\rho} \beta_j^4 \sinh(\beta_j L)$$

$$F_{22} = \beta_j^3 \sin(\beta_j L) - \beta_j^3 \sinh(\beta_j L) + \frac{M_L}{\rho} \beta_j^4 \cos(\beta_j L) - \frac{M_L}{\rho} \beta_j^4 \cosh(\beta_j L)$$

From equation (2.14), equating the $\det |F|$ to zero will give a transcendental equation as follows :

$$(1 + \cos(\beta_j L) \cosh(\beta_j L)) - \frac{M_L \beta_j}{\rho} (\sin(\beta_j L) \cosh(\beta_j L) - \cos(\beta_j L) \sinh(\beta_j L)) -$$

$$\frac{J_L \beta_j^4}{\rho} (\sin(\beta_j L) \cosh(\beta_j L) + \cos(\beta_j L) \sinh(\beta_j L)) + \frac{M_L J_L \beta_j^4}{\rho} (1 - \cos(\beta_j L) \cosh(\beta_j L)) = 0 \quad (2.15)$$

Putting the value of system parameter From table 2.1, find out the value of β_j from equation (2.15) and substitute in equation (2.14) to calculate the constant value defined in equation 2.13.

Applying Eulers Lagrange equation,

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{q}_i} \right] - \frac{\partial L}{\partial q_i} = W_i \quad (2.16)$$

where, $L = E_k - E_p$ is a lagrangian.

we get the dynamic equations of motion for a single-link flexible arm, which can be written in the closed form as:

$$M(q)\ddot{q} + h(q, \dot{q}) + Kq = \tau \quad (2.17)$$

where, $q = (\theta, \partial)^T$ represent the state vector of the model. In this thesis we consider two modes of vibration for single-link flexible robot manipulator hence six state have to be considered. The states are gives as

$$q = [\theta, \partial_1, \partial_2, \dot{\theta}, \dot{\partial}_1, \dot{\partial}_2]$$

M is a positive-definite symmetric inertia matrix.

h is a vector of a centripetal forces.

K is the diagonal stiffness matrix.

τ is a column vector consisting of control torque at the joint location.

The coefficients of mass matrix (M) are:

$$M_{11}(\partial) = J_0 + J_L + M_L L^2 + M_L (\phi_e^T \partial)^2 \quad (2.18)$$

$$M_{1j} = M_L L \phi_{j-1,e} + J_L \phi'_{j-1,e} + \sigma_{j-1}, j = 2, \dots, m+1 \quad (2.19)$$

$$M_{ii} = m_b + M_L \phi_{j-1,e}^2 + J_L \phi_{j-1,e}'^2, \quad i = 2, \dots, m+1 \quad (2.20)$$

$$M_{ij} = M_L \phi_{j-1,e} \phi_{j-1,e} + J_L \phi'_{j-1,e} \phi'_{j-1,e}, i = 2, \dots, m+1 \quad (2.21)$$

where,

$$\phi_e^T = \phi_e^T|_{x=l} = [\phi_1 \dots \phi_m]|_{x=l} \quad \phi_{ie} = \phi_i(l) \quad (2.22)$$

$$\sigma_i = \rho AL^2 \int_0^l \phi_i(x) x dx, \quad i = 1, 2, \dots, m+1 \quad (2.23)$$

The non-linear terms h_1 and h_2 known as Centrifugal forces respectively are given below:

$$h_1 = 2M_L \dot{\theta}(\phi_e^T \partial)(\phi_e^T \dot{\partial}) \quad (2.24)$$

$$h_2 = -M_L \dot{\theta}^2(\phi_e^T \phi_e^T) \partial \quad (2.25)$$

Equivalent spring constant matrix K is given below as:

$$K = \text{diag}\{0, w_1^2 m_b, w_2^2 m_b\} \quad (2.26)$$

where,

m_b is the link mass.

I_0 is the joint actuator inertia.

J_0 is the link inertia relative to the joint.

M_L and J_L is load mass and load inertia, respectively.

Physical system parameters of a single-link flexible robot manipulator system:

Table 2.1: Physical system parameters of a flexible robot manipulator

Parameter	Symbol	Value	Units
Length of link	L	0.22	m
Mass of link	m_b	0.065	kg
Mass per unit length of link	ρ	0.2	kg/m
Link inertia related to joint	J_0	0.0083	kgm^2
Moment of inertia of joint actuator	I_0	$6.28e^{-6}$	kgm^2
Tip inertia	J_L	0.005	kgm^2
Tip mass load	M_L	0.1	kg
Elasticity of link	EI	1	Nm^2

2.2.2 Actuator Dynamics

The actuator dynamics constitute an important part of the flexible robot dynamics. It is used to drive the link of flexible robot. The actuator consider is a dc motor and placed at the hub of link connected through gear-box for safe operation of the link. The control signal is feed to motor from the amplifier and harmonic drive limits the speed of operation. Figure 2.3 shows the harmonic drive is connected in between DC motor and flexible robot.

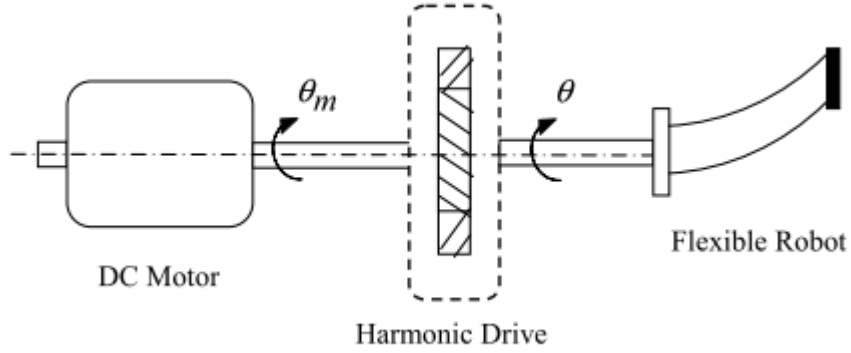


Figure 2.3: DC motor connection to flexible robot through harmonic drive.

Let T_m, T_1, T_2 be the torque developed by the motor, torque at the motor shaft, torque transmitted to the load and T_L load torque. θ_m and θ_L be the position of motor at motor shaft and load shaft respectively. J_m is the inertias of motor and J_L is the inertias of load. R_a, L_a, K_b, K_t and N_r are the armature resistance, inductance, back emf constant, motor torque constant and gear ratio respectively.

By applying KVL, the voltage equation for the armature circuit defined as:

$$u = L_a \frac{di_a}{dt} + R_a i_a + e_b \quad (2.27)$$

where, $e_b = K_b \dot{\theta}_m$ is the back-emf generated in the armature circuit. Given the motor voltage u and the current i_a flow through the armature circuit and develops electro-magnetic torque as $T_m = K_t I_a$.

The harmonic drive is integrated with motor shaft and the flexible link is mounted on the harmonic drive. The ratio of speed can be given as follows:

$$N_r = \frac{\theta_m}{\theta_L} = \frac{T_2}{T_1} \quad (2.28)$$

The torque balance equation can be written as follows:

$$T_m = J_m \ddot{\theta}_m + T_1 \quad (2.29)$$

and torque transmitted to load is given as follows:

$$T_2 = J_L \ddot{\theta}_L + T_L \quad (2.30)$$

From equations (2.28)-(2.30), the load torque can be given as

$$T_L = N_r T_m - J_h \ddot{\theta} \quad (2.31)$$

where, hub inertia of the flexible link robot $J_h = J_L + N_r^2 J_m$ is the total inertia referred to the load side of the motor. Substituting for T_m in equation (2.31), we can get expression for load torque as follows:

$$T_L = N_r K_t i_a - J_h \ddot{\theta}_L \quad (2.32)$$

The torque T_L developed as given in equation (2.32) is used to drive the flexible link through the speed reducer. The actuator current i_a has to consider as one of the states in the model obtained using AMM discussed in above section. Thus we have to consider one more state in the model obtained using AMM in equation (2.17). Thus the actuator dynamics increases the system order by one. Thus we can rewrite the above model equation (2.17) as follows:

$$M(q)\ddot{q} + h(q, \dot{q}) + Kq = \tau \quad (2.33)$$

where,

$$q = [\theta, \partial_1, \partial_2, \dot{\theta}, \dot{\partial}_1, \dot{\partial}_2, i_a]$$

Parameters of Actuator:

Table 2.2: Parameter of Actuator

Parameter	Symbol	Value	Units
Stiffness constant	k_s	2	Nm/rad
Gear ratio	k_g	60	NA
Torque Constant	k_m	0.00767	$V/(rad/sec)$
Motor Constant	k_m	0.00767	Nm/Amp
Motor Resistance	R_m	2.6	Ω

2.3 Chapter summary

In this chapter the experimental diagram of two link flexible robot manipulator with their major components has been discussed. In this project work, we have considered only single link of flexible robot manipulator. Next it is followed by modelling of a single link flexible Robot manipulator using AMM in which actuator dynamics are also taken into consideration. Considering the actuator dynamics increases the order of the system by one.

Chapter 3

System identification of a flexible robot manipulator

In this chapter identification of a flexible robot manipulator is done by considering different types of model representation. In section 3.1, gives the definition of system identification and the need of system identification for process modelling the system along with the advantage of system identification. In section 3.2 a general structure of linear model identification is given. The linear model i.e OE and ARX model are discussed with there structure. In section 3.3, the nonlinear model identification by using NARX model is described and finally in section 3.4 simulation results are discuss.

3.1 Introduction

System identification is the process of developing or representation of mathematical model of a physical system using experimental data. Modeling from first principles, requires an in-depth knowledge of the system. System identification methods can handle a wide range of system dynamics without knowledge of the actual physics of system. This is one of the advantage of using the system identification method to represent the model of physical systems. For representation of flexible robot manipulator, we consider first the linear model and then we go for nonlinear model.

3.2 Linear model identification

For modelling of unknown system different parametric model structures can be selected. Parametric models describe the true process behavior exactly with finite number of parameter and the model is represented in terms of differential equations and transfer functions. The parametric model structure is also known as a black-box model.

General-linear model equation is defined as:

$$y(k) = q^{-k}G(q)u(k) + H(q)e(k) \quad (3.1)$$

where, $u(k)$ and $y(k)$ are the input and output of the system respectively.

$e(k)$ is zero-mean white noise, or the disturbance of the system.

$G(q)$ is the transfer function of the deterministic part of the system.

$H(q)$ is the transfer function of the stochastic part of the system.

The general-linear model structure, shown in Figure 1, By choosing proper $A(q), B(q), C(q)$

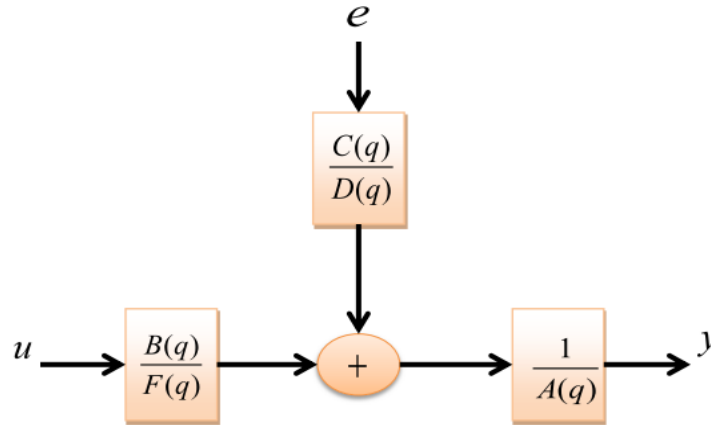


Figure 3.1: General-Linear Model Structure

and $D(q)$ polynomial general-linear model structure divided into different structure discuss in below.

3.2.1 Output Error(OE) model

The Output-Error(OE) model structure describes the system dynamics separately. No parameter are used for modeling the disturbance characteristics.

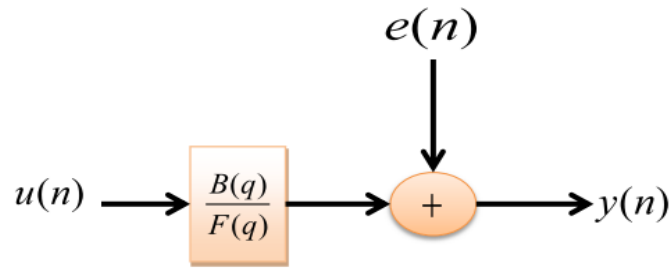


Figure 3.2: Block diagram of a OE model

3.2.2 Auto Regressive Exogenous (ARX) model

The estimation of an ARX model is the most efficient of the polynomial estimation methods because it is the result from solving linear regression equations in analytic form. The

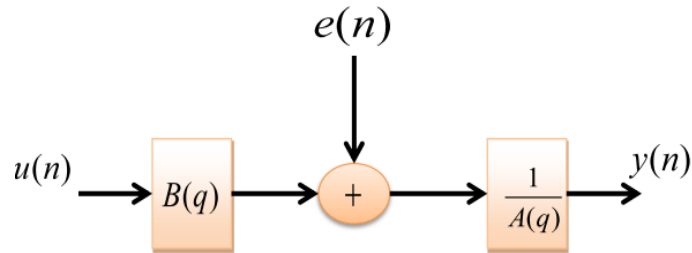


Figure 3.3: Block diagram of a ARX model

structure of the output error (OE) model is similar to that of the ARX model except that the model output in the OE Model form is a function of the past inputs and the past model outputs, while the model output in the ARX form is a function of the past inputs and past process outputs. From identification results figure (3.6 and 3.7), it is seen that the linear model (OE and ARX model) is not good enough to identify the system having nonlinear nature. When there is a nonlinear parameter in the system then identification by using OE or ARX model can not gives the complete characteristics of the nonlinear system. That means the nonlinear term present in the system is not modeled properly. Because of this we consider nonlinear model identification.

3.3 Nonlinear model identification

3.3.1 Nonlinear Auto Regressive Exogenous(NARX) model

To identify the non-linear and time-varying system the NARX model is used. The general form of single-input single-output NARX model to represent the nonlinear system can be defined as

$$y(k) = f(y(k-1), \dots, y(k-n_y), u(k-1), \dots, u(k-n_u)) + \xi(k) \quad (3.2)$$

where, the function $f(\cdot)$ represents the cross product and higher order polynomial terms. The degree of the power terms in $y(k)$ and $u(k)$ is referred to as the degree of nonlinearity. and n_y, n_u are the maximum delay in the output and input respectively and $\xi(k)$ is the white noise signal. Assuming $f(\cdot)$ as a polynomial of degree l gives the following representation of model.

$$y(k) = \theta_0 + \sum_{i_1=1}^n \theta_{i_1} x_{i_1}(k) + \sum_{i_1=1}^n \sum_{i_2=i_1}^n \theta_{i_1 i_2} x_{i_1}(k) x_{i_2}(k) + \dots + \xi(k) \quad (3.3)$$

where $n = n_y + n_u$ and

$$x_1(k) = y(k-1), x_2(k) = y(k-2), \dots, x_{n_y}(k) = y(k-n_y),$$

$$x_{n_y+1}(k) = u(k-1), x_{n_y+2}(k) = u(k-2), \dots,$$

$$x_{n_y+n_u}(k) = u(k-n_u)$$

The polynomial coefficients θ_0 are unknown parameters to be determined for the given input-output data and x_i are delayed input and output terms. Above equation can be written in the linear regression function form as

$$y(k) = \sum_{i=0}^M p_i(k) \theta_i + \xi(k) \quad (3.4)$$

where the number of unknown parameter M depends on the value of n_y, n_u and l , whereas p_0 and $p_i(k)$ are regressors as defined by x_i . For a polynomial function representing quadratic non-linearity of order l , the M monomials are given as

$$M = 1 + n + \frac{n(n+1)}{2}$$

$$p_1(k) = y(k-1), \dots, p_{n_y+1}(k) = u(k), \dots, p_{n+1}(k) = y^2(k-1), \dots, p_{n+}(k) = u^2(k-1), \dots,$$

$$p_M(k) = y(k-n_y)u(k-n_u).$$

The incorporation of time-varying dynamics demands an adaptive model with varying parameters. In this regard, the parameter are iteratively updated by considering input-output

data over different time intervals. Over each interval, the parameter estimation is carried out using least-square method. The identification accuracy, in terms of replicating the experimental dynamic is found to be much better for the adaptive model than the fixed structure model.

3.4 Identification results

For identification, the SLFM is excited with band-limited white noise (noise power = 20 watt and sampling time = 0.001 sec) as shown in figure 3.4 and the corresponding output in term of rotation angle is shown in figure 3.5. The experiment is performed for 2 sec with sampling time 0.001 sec i.e a record of 2000 experimental samples are considered. Firstly identification is done by using linear model i.e OE and ARX model. The order of linear model i.e OE model and ARX model is taken as two. The simulation results of linear model (OE and ARX) identification is depicted in figure 3.6 and 3.7. From the output of the estimated model it is seen than the linear model is not sufficient to represent the nonlinear and time varying system. Hence we go for nonlinear model identification i.e NARX model identification. For identification NARX model, the order and number of delayed terms in the input and output are considered as 2 and 3 ($n_u = n_y$) respectively. Simulated results of the identified NARX model in figure 3.8 depicts that the estimated NARX model is also not good enough to represent the SLFM system as the NARX model used is time-invariant in nature. To solve this difficulty we go for adaptive NARX model identification. Simulation results depicted in figure 3.9 shows that the adaptive NARX model gives correct representation of SLFM dynamics. The error between experimental output and estimated model output is quite low (figure 3.9).

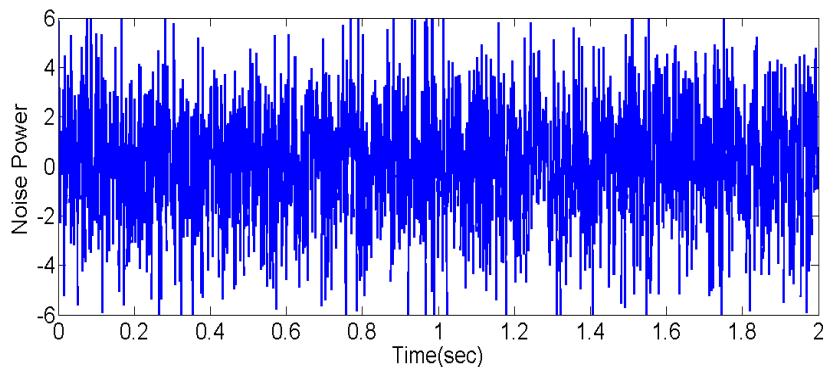


Figure 3.4: Experimental input data

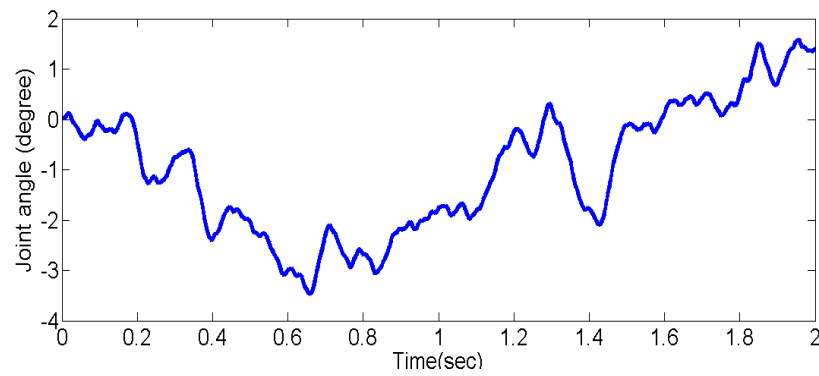


Figure 3.5: Experimental output data

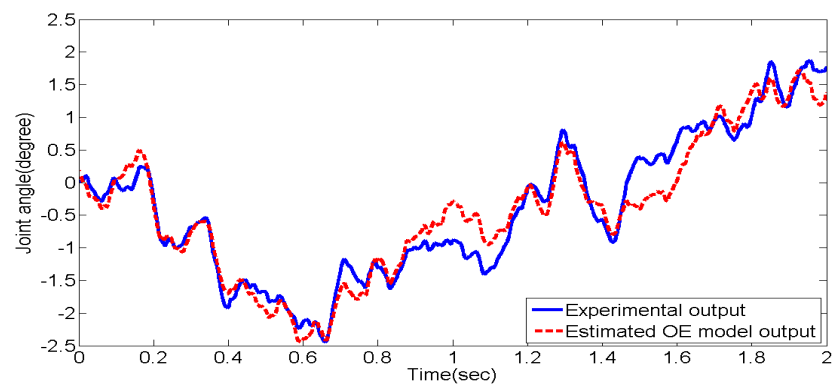


Figure 3.6: Experimental output Vs the simulated output of the identified OE model

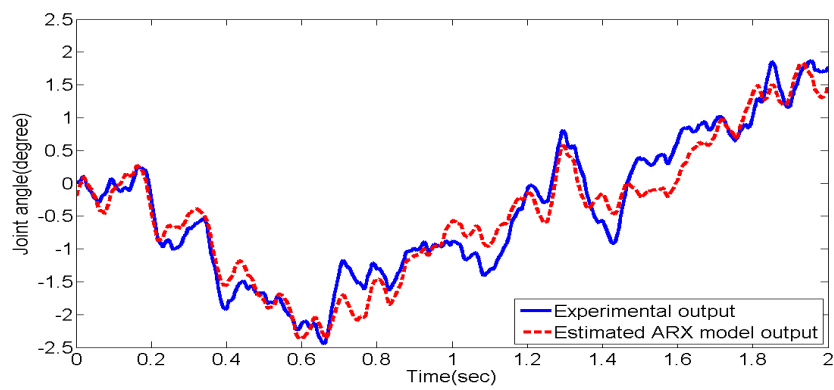


Figure 3.7: Experimental output Vs the simulated output of the identified ARX model

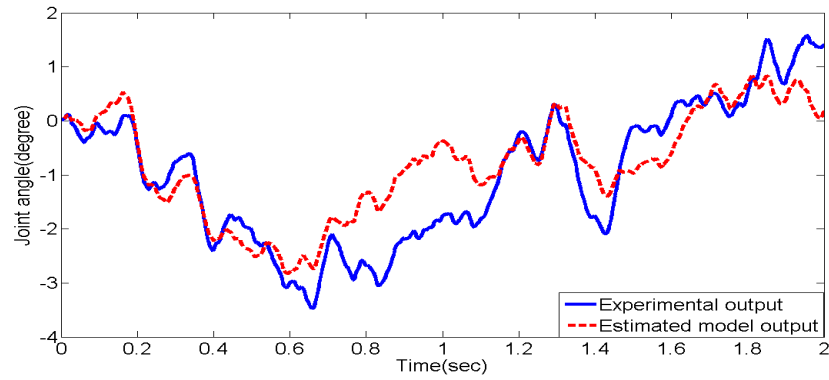


Figure 3.8: Experimental output Vs the simulated output of the identified NARX model

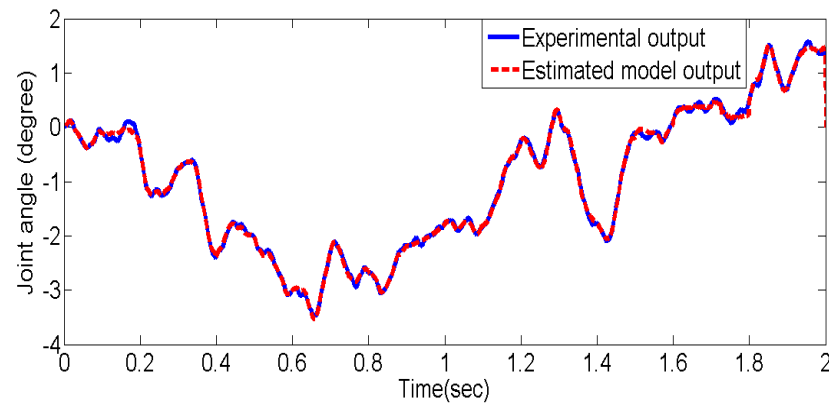


Figure 3.9: Experimental output Vs the simulated output of the identified adaptive NARX model

3.5 Chapter summary

This chapter identification of SLFM system by using linear model as well as nonlinear model has presented. In linear model identification OE and ARX model id identified. From simulation results it is seen that linear model is not gives correct representation of nonlinear system. In nonlinear identification time varying NARX model is identified. The time invariant NARX model also not suitable to represent the highly nonlinear system. Hence for identification of highly nonlinear system an adaptive NARX model gives correct representation of nonlinear system.

Chapter 4

Controller Design

In this chapter, An Iterative Learning based Control strategy is designed for a flexible link robot manipulator. In section 4.1, Iterative learning controller is discuss by highlighting the advantage of using iterative learning controller. In section 4.2 an Adaptive iterative learning controller is presented. In section 4.3, the controller is designed based on an identified adaptive NARX model and finally in section 4.4 simulation results are discussed.

4.1 Iterative Learning Controller

4.1.1 Introduction

Iterative Learning Control (ILC) is a control technique, designed for the system showing repetitiveness in its operations. Iterative learning based control technique is used to enhance tracking performance, by using the error inputs obtained from each trial.

4.1.2 Advantage of Iterative Learning Controller

Some advantages of using Iterative learning based control technique are as follows:

- Simplicity of the structure.
- Good output tracking.
- Model-independent design.
- Improves transient response and tracking performance of processes or system that executes the same operation over and over.

Thus ILC possesses greater advantages over other traditional controller design techniques employed for the plants which are having uncertainty in the system.

4.1.3 Design of Iterative Learning Controller

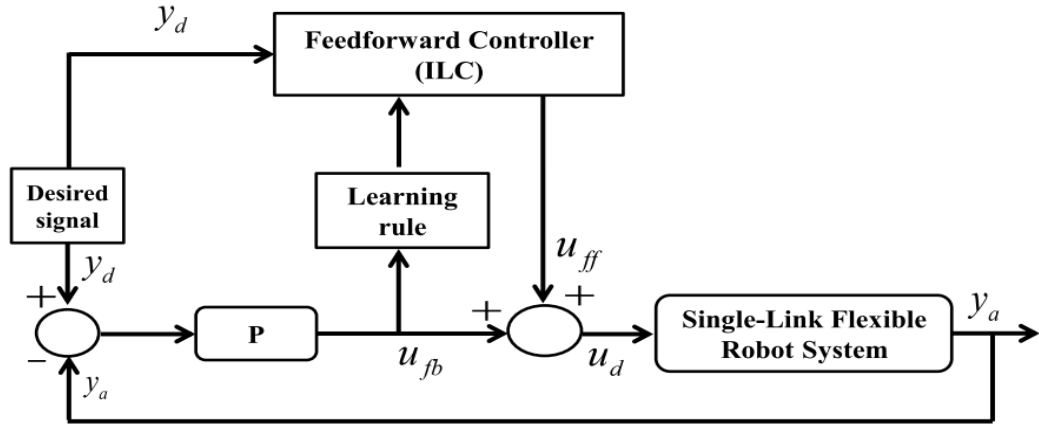


Figure 4.1: Structure of ILC controller for a Flexible robot manipulator

In this section, Iterative learning based control technique is used to enhance tracking performance of the single-link robotic manipulator, by using the error inputs obtained from each trial. The ILC has been implemented using the least square algorithm. It comprises feed-forward learning controller and a linear forward controller (Figure 4.1). In linear forward path PID_1 controller is used for stabilizing the system and the feed-forward path incremental structure of ILC controller is used for tracking purpose. Hence the structure of ILC controller shown in figure 4.1 is modified to get the incremental structure of ILC controller. Figure 4.2 shows the incremental structure of ILC controller. The advantage of using incremental structure is to avoid problems of filtering cause by measurement noise, obtained from sensor. The following cost function is used to find out the desired control input through iteration. For finding desired control signal we have minimize the cost function defined below.

$$J_t = \frac{1}{2} \sum_{k=1}^{\infty} \| u_d(t) - u_{ff}^k(t) \|^2 \quad (4.1)$$

Learning Rule:

$$u_{ff}^{k+1}(t) = u_{ff}^k(t) - \lambda \frac{\partial J_t}{\partial u_{ff}^k} \quad (4.2)$$

$$u_{ff}^{k+1}(t) = u_{ff}^k(t) + \lambda(u_d(t) - u_{ff}^k(t)) \quad (4.3)$$

$$u_{ff}^{k+1}(t) = u_{ff}^k(t) + \lambda u_{fb}^k(t) \quad (4.4)$$

where, λ is the learning gain and

$$u_d(t) = u_{ff}^k(t) + u_{fb}^k(t) \quad (4.5)$$

For designing incremental structure of ILC controller we modified the block diagram of ILC

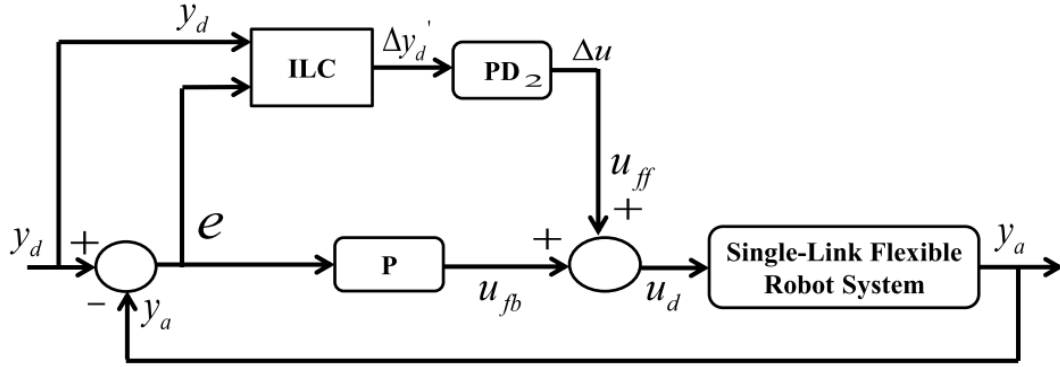


Figure 4.2: Modified Structure of ILC controller for a Flexible robot manipulator

controller in figure 4.1 and it is represented in figure 4.2. The design of ILC controller is carried out in two phases. In the first phase, an ILC update law is carried out to yield the ideal input and output signals of the overall ILC augmented control system. In the second phase, signals 'e'(error) and ' Δu '(the change in reference input) are used to calculate the tuning parameters of PD_2 controller by using a standard least-squares (LS) algorithm. The update law is given as

$$\Delta u(t) = k_P e(t) + k_D \frac{de(t)}{dt} \quad (4.6)$$

$$\Delta y_{d,i+1}(t) = \Delta y_{d,i}(t) + \lambda e_i(t-1) \quad (4.7)$$

where λ is the learning gain.

A PD-type update law [11] has been adopted for ILC. In this update law, the change in the reference input is directly proportional to the error and its derivative term in each trial. Once

the actual error (e) becomes very small (within bound of 0.002 m) the ILC stops updating. The ideal input 'e' and output ' Δu ' for a cycle of the reference signal is now available for the next phase.

Using e and Δu , parameter θ of PD_2 is determined by using least square method as

$$\Delta u(t) = \varphi^T(t)\theta \quad (4.8)$$

where,

$$\theta = [k_{P2} \ k_{D2}]^T \text{ and } \varphi^T(t) = [e(t) \ \frac{de(t)}{dt}]$$

k_{p2} and k_{D2} are the tuning parameters of the PD_2 controller.

Using LS algorithm, we have

$$\theta = (\phi^T \phi)^{-1} \phi^T U \quad (4.9)$$

where,

$$\phi = [\varphi^T(1) \ \varphi^T(2) \ \dots \ \varphi^T(N)]^T$$

$$U = [\Delta u(1) \ \Delta u(2) \ \dots \ \Delta u(N)]^T$$

and N is the number of data used in estimation.

4.2 Adaptive Iterative Learning Controller

The update law of iterative learning controller given in equation 4.7 is the same used here for designing of an adaptive iterative learning controller. In this equation, a constant value of learning gain λ is used. For designing of AILC controller, learning gain must be change with respect to iteration. The following equation from [24] is used to make learning gain adaptive.

$$\lambda(k+1) = \lambda(k) + \beta \|e(k)\|^2 \quad (4.10)$$

where, scalar β value is taken as unity.

The structure of AILC is same as that of ILC controller with small change in learning gain. Instead of constant learning gain, we used the learning gain given in equation 4.7.

4.3 Controller design based on identified model

4.3.1 Iterative Learning controller based on identified NARX model

Figure 4.3 shows the structure of ILC controller designed using an identified adaptive NARX model. Identification of flexible link robot manipulator is done by using adaptive NARX model presented in chapter 3. Identified adaptive NARX model is used for design of ILC controller because of some advantages like it does not require a priori complete knowledge of system and it is independent of parameter variation. The model is identified online and used in controller design.

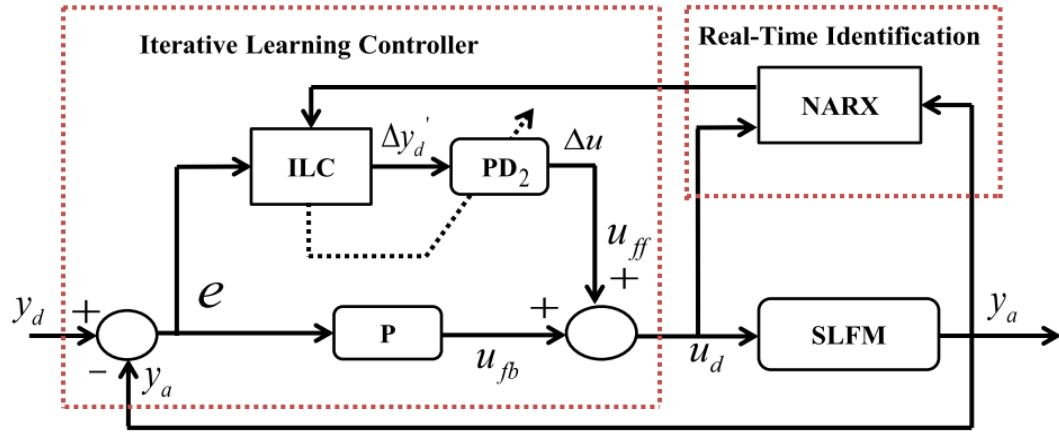


Figure 4.3: Structure of adaptive NARX model based iterative learning controller for SLFM

4.3.2 Adaptive Iterative Learning controller based on identified NARX model

For designing of AILC controller based on an identified adaptive NARX model, the structure is same as that of designing ILC controller based on an identified adaptive NARX model. But here learning gain used is from equation 4.10.

4.4 Simulation Results and Discussions

4.4.1 Simulation results of ILC controller

The designed ILC controller is simulated first based on the existing mathematical model given in chapter 2 and then on identified adaptive NARX model. The proposed ILC controller is tested first for repetitive signal i.e sinusoidal signal and then for fixed signal i.e step signal.

Learning rate $\lambda = 1.9$ chosen for simulation because of lesser mean square error.

Table 4.1: Mean square error for different learning rate

Learning rate (λ)	Mean square error (MSE)
0.1	0.6266
0.5	0.3092
1.0	0.0236520
1.5	0.0004569
1.9	0.0000661
2.5	0.0280

The sinusoidal signal used is having a frequency 3 rad/sec and amplitude 1 degree. Figure 4.4 shows that the desired output versus actual output for the first iteration. As the iteration increases the output of SLFM matches with the desired trajectory. The output of fifth iteration is shown in figure 4.5, The simulated output does not completely converges the actual output. The actual output converges to the desired trajectory in the tenth iteration (figure 4.6).

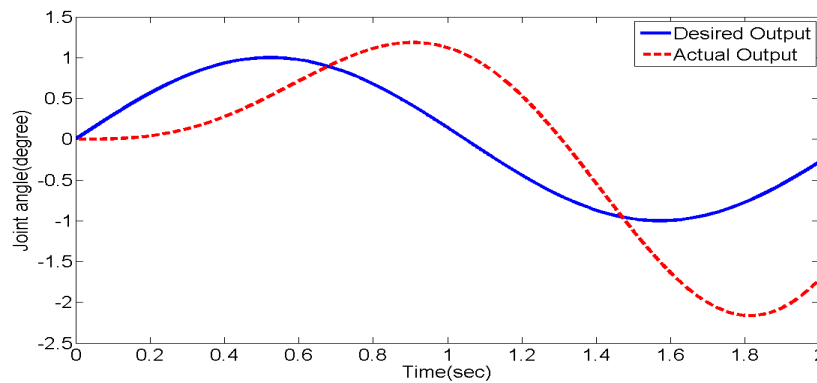


Figure 4.4: Desired output Vs Actual output in the first iteration.

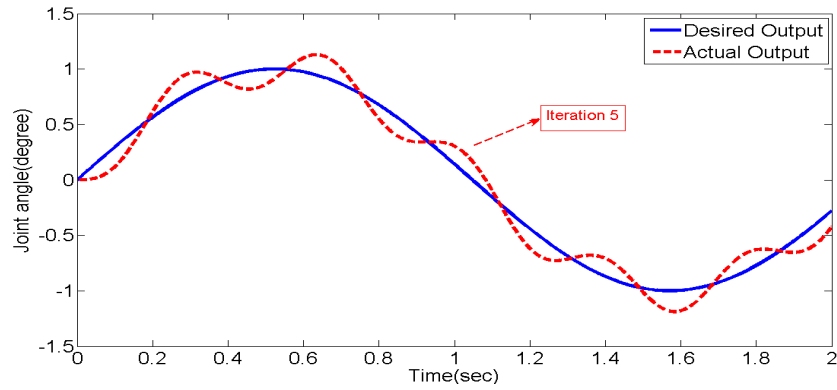


Figure 4.5: Desired output Vs Actual output in the 5 iteration.

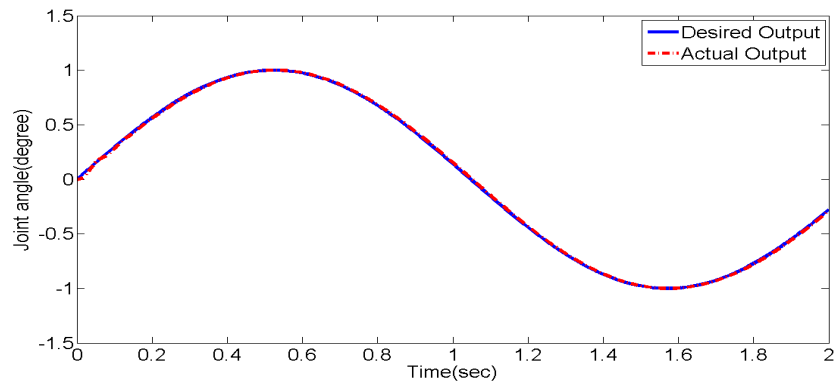


Figure 4.6: Desired output Vs Actual output in the tenth iteration.

For determining the effectiveness of the controller in tracking a desired signal, a step signal is used as a desired trajectory having amplitude of 1 degree and at step time 0.5 and 1 sec shown in figure 4.7. Figure 4.7 shows that the desired output versus actual output for the first iteration. As the iteration increases the output of SLFM matches with the desired trajectory. The output of fifth, seventh and ninth iteration shown in figure 4.8 is having some initial oscillation but after some time it converges to desired value. At the tenth iteration the actual output converges to the desired trajectory without any oscillation shown (figure 4.9).

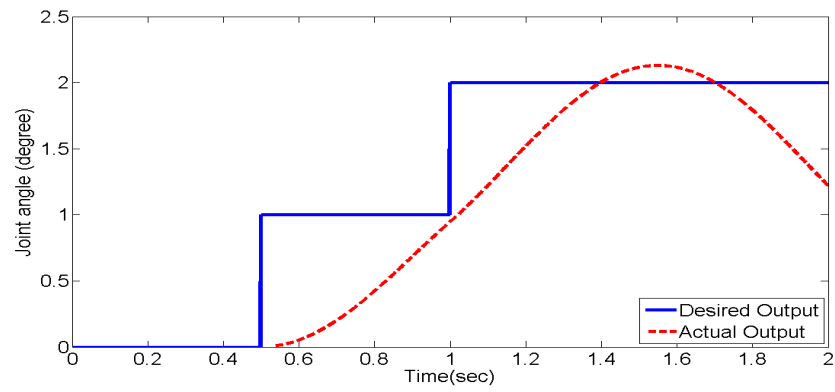


Figure 4.7: Desired output Vs Actual output in the first iteration.

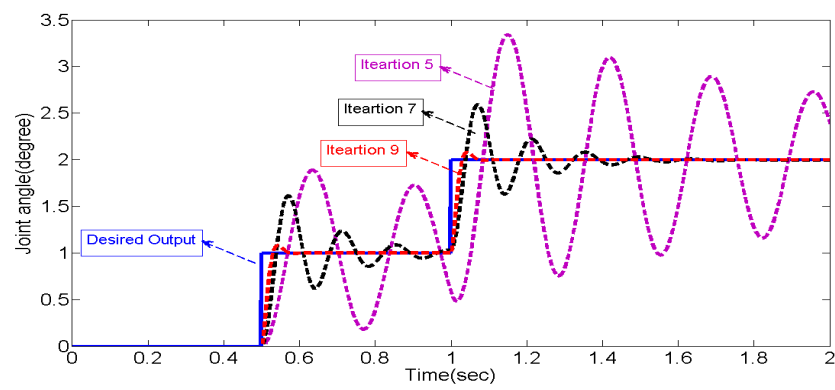


Figure 4.8: Desired output Vs Actual output in the 5,7 and 9 iteration.

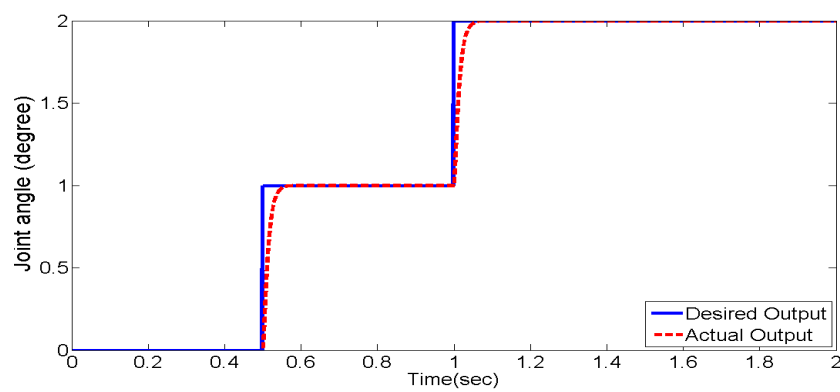


Figure 4.9: Desired output Vs Actual output in the tenth iteration.

4.4.2 Simulation results of AILC controller

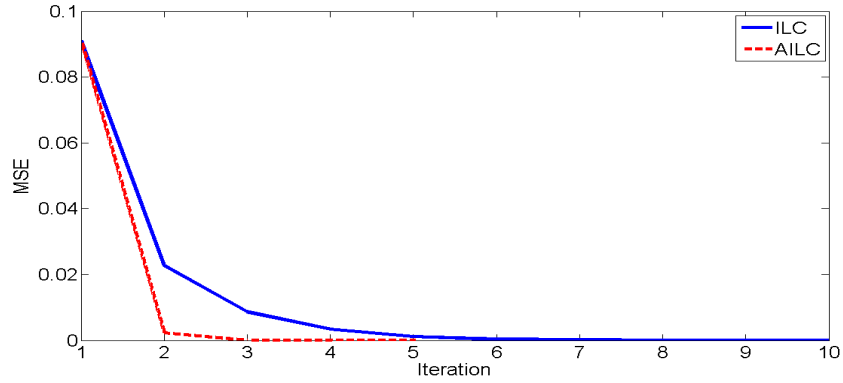


Figure 4.10: Iteration variation of mean square error(MSE)of ILC controller Vs AILC controller.

From figure 4.10, it is seen that mean square error is reduced at high rate as compared to the MSE of ILC controller. Also by using AILC controller, the number of iteration required to converge the actual output with desired output is less than ILC controller.

4.4.3 Simulation results of ILC controller based on an identified adaptive NARX model

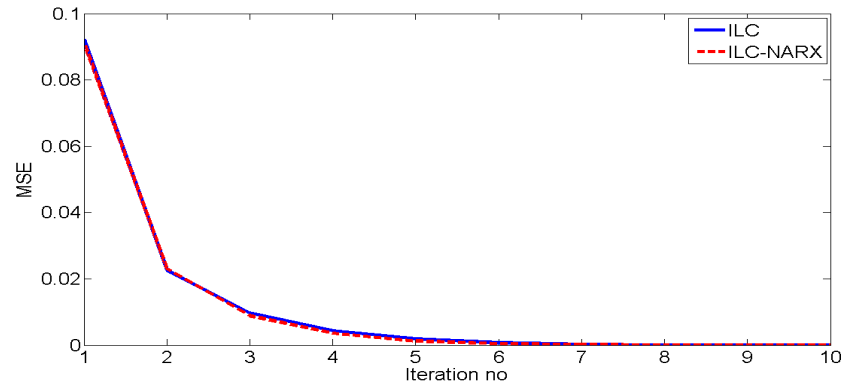


Figure 4.11: Iteration variation of mean square error (MSE) of ILC controller based on an identified adaptive NARX model without considering actuator dynamics.

Figure 4.11 gives the MSE plot versus iteration of ILC controller when it is designed using an identified adaptive NARX model without considering actuator dynamics. From figure it is seen that there is a small improvement in output by using identified adaptive NARX model for designing ILC controller. But when actuator dynamics are included then order of system

is increased by one means the mathematical complexity in the model increases and model becomes highly nonlinear. The adaptive NARX model is independent of complexity in the model. Figure 4.12 shows that the ILC controller designed based on an identified adaptive NARX model with considering actuator dynamics gives improved results as compared to without considering actuator dynamics.

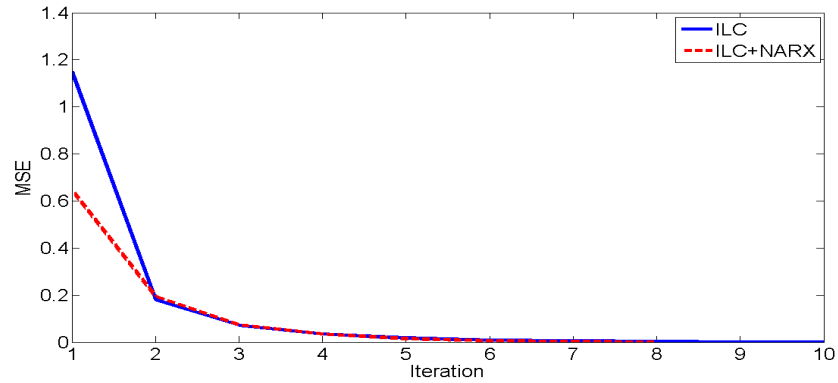


Figure 4.12: Iteration variation of mean square error (MSE) of ILC controller based on an identified adaptive NARX model with considering actuator dynamics.

4.4.4 Simulation results of AILC controller based on an identified NARX model

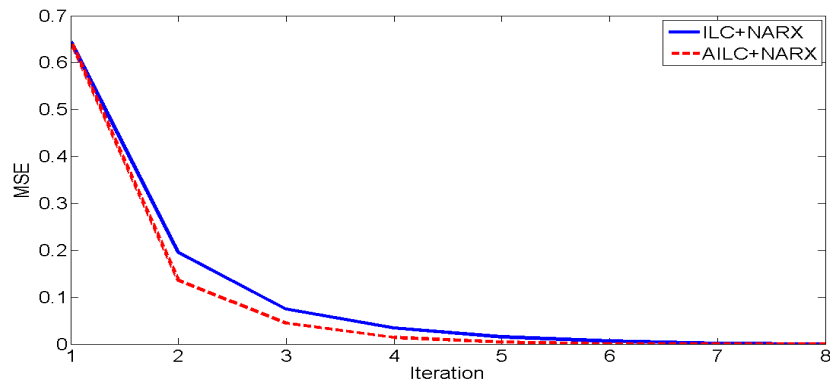


Figure 4.13: Iteration variation of mean square error (MSE) of AILC controller based on an identified adaptive NARX model.

Figure 4.13 gives the mean square error plot comparison between ILC controller and AILC controller designed based on an identified adaptive NARX model with considering the actuator dynamics. From simulated results it is seen that AILC controller gives better results

than ILC controller when both are designed based on an identified adaptive NARX model.

4.5 Conclusion of Simulation Results and Chapter summary

For designing an ILC controller, In linear forward path simple P controller having gain $k_p = 1$ is used to stabilize the system and in feed-forward path PD_2 type controller law with the gain parameters ($k_p = 0$ and $k_d = 0$) chosen initially, This setting is unable to provide good results for both the sinusoidal and a step input. This can be easily understood from the Figure 4.4 and Figure 4.7, which show the actual joint position versus desired joint position respectively. Using Iterative Learning based control technique in first phase the output comes out to be stabilized. For perfect tracking we have to update PD controller law by using the data of error and the change in reference input to tune the existing PD controller parameters. Figure 4.5 and figure 4.8 shows that the actual output start converging to desired output with increase in the iteration. From simulation results it is seen that after tenth iteration parameter of PD controller is tuned such that actual output of ILC controller completely track the desired output.

Next, ILC controller is designed based on an identified adaptive NARX model. Identification of SLFM system is done real-time and identified adaptive model used for ILC controller design. From simulation results in figure 4.12, it is seen that as complexity increases the performance of ILC controller based on an identified adaptive NARX model increases. The constant learning gain is used ($\lambda = 1.9$) because it gives very small value of mean square error. Lastly the learning rate is made adaptive to design adaptive ILC controller. From figure 4.10, AILC controller gives better results than ILC controller. Also AILC controller designed using an identified adaptive NARX model gives better results as shown in figure 4.13 as compared to ILC controller based on an identified adaptive NARX model.

In this chapter, introduction of Iterative Learning controller and its advantages, and the design of iterative learning controller is discussed with update laws. The design of ILC controller is extended to design ILC controller based on an identified adaptive NARX model. Also AILC controller is designed by making learning rate adaptive. Lastly simulation results are presented for a single link flexible robot manipulator. The results obtained, verifies that the AILC controller design based on an identified adaptive NARX model is better than the ILC controller design based on an identified adaptive NARX model.

Chapter 5

Conclusion and Future work

5.1 Conclusion

For designing a proper controller for tracking a desired trajectory, the single-link flexible manipulator system is first identified by using adaptive NARX model from experimental data. Simulation results shows that the identified adaptive NARX model gives good representation of SLFM dynamics and avoids complexity resulting from the use of partial differential equation based model.

Based on the identified adaptive NARX model, we propose the incremental structure of ILC controller for a single-link flexible manipulator and it is extended to AILC controller. The proposed AILC based approach is found to be quite effective in tracking a desired trajectory over a definite time interval.

5.2 Suggestions for future work

The proposed work in this thesis is for single link flexible robot manipulator which can be extended in future perspective for multi link flexible robot manipulator. Besides this, future work in this direction is aimed at:

- Modeling techniques can be extended to two link flexible robot manipulator with incorporating actuator dynamics in assumed mode model.
- The ILC and AILC controller designed technique based on an identified adaptive NARX model can be used for designing controller for different payload condition.

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